The Effect of Clustering Coefficient and Node Degree on The Robustness of Cooperation

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Abstract—This paper explores the robustness of cooperation in a spatially organised population of agents participating in the N-player prisoner's dilemma. The agents are placed on graphs exhibiting different properties and the relationship between these properties and the robustness of cooperation is explained. In particular, this paper analyses the effect the clustering coefficient and the average node degree has on cooperation. In addition to theoretical analysis, rigorous experiments, involving the creation of graphs exhibiting certain desirable properties, are undertaken to explore the effect of the graph properties on the ability of cooperation to resist invasion. Both the theoretical and the experimental results show that when the average degree is high, the population loses the ability to maintain cooperation in the presence of defectors. However, for graphs with lower average node degree, a higher clustering coefficient will guarantee a relatively high cooperation rate.

Keywords—*Evolutionary game, Prisoner's Dilemma, Cooperation*

I. Introduction

The graph topology governing the structure of a population of agents can influence the emergence of cooperation in evolutionary games [1], [2], [3], [4]. Previous research on different graph topologies has shown that the type of topology may exert different levels of influence on the evolutionary dynamics of the population; research has included exploring lattice grids [5], [6], [7], [8], [9], Newman-Watts small-world network [10], [11], [12], and scale-free networks [2], [4], [13], [14]. However, a graph of a certain topology is defined by several parameters or properties (for example, a lattice graph has a certain degree; and many different lattices can be created each with their own degree). Although, the relationship between certain properties of a graph and their influence on evolutionary dynamics is still not known, many researchers believe certain parameters of the graph are critical to the emergence of cooperation, such as the average degree [10], [15], and the clustering coefficient [16], [17].

In this paper we explore the relationship between certain graph properties and the robustness of cooperation in a population of agents. In order to explore this relationship, we designed two graph generation algorithms in order to generate graphs which exhibit a set of desirable features, namely: a pre-designed size, a pre-defined number of edges, and a specific clustering coefficient. By running simulations on graphs of the same size but with different clustering coefficients, we were able to observe and analyse the effect the clustering coefficient has on cooperation. The simulations show that when one player in a connected, fully cooperating society is mutated to defect, the entire society will turn to defection quickly if the average degree is high. However, when the number of edges is relatively low, the cooperation rate is linearly proportional to the clustering coefficient in the graph.

The remainder of the paper is as follows: the next section, Section 2, presents a short overview of previous research; Section 3 presents algorithms for generating graphs exhibiting a specific clustering coefficient; Section 4 presents our analysis of the effect that different graph topologies may have on cooperation; Section 5 presents the experimental set up and results; finally, Section 6 presents the conclusions and possible further work.

II. Background

Adopting natural selection approaches in evolutionary games leads to those agents obtaining higher payoffs being selected at the expense of less fit agents. For agents positioned on a complete graph, defectors obtain a higher payoff by exploiting cooperators and are more likely to invade and eliminate the cooperators. The result in this scenario can be calculated using replicator dynamics [18]. However, for real world social networks, the graph is not fully connected but instead can exhibit variance in degree, clustering coefficient and other properties. To understand the individual agent's behaviour in such graphs, evolutionary game simulations are usually adopted; often times these simulations show that co-operators have a greater chance to survive and be robust in the presence of defectors.

Nowak concluded that without any mechanism that could benefit cooperators, natural selection will always favor defectors [1]. The effect of the graph topology, which he termed "Network Reciprocity", is one of the five mechanisms that could help to maintain cooperation in the face of potential invasion of defectors. There are experiments supporting the idea that the population structure of the agents can have a significant effect on the emergence of cooperation [2], [4]. Nowak and May introduced a spatial structure on agents undergoing evolution where fitness was determined by their score in the social dilemma game [5], [6], [7], [8]; their experiments showed that a group of cooperators in the spatial structure could maintain cooperation and avoid exploitation by defectors. However, Hauert also showed, in snowdrift games,

that the spatial structure is more often detrimental to the emergence of cooperation [19].

Many researchers moved from exploring the lattice grid to considering more complex graphs such as small-world networks and scale-free networks. For example, Santos and Pacheco found that the cooperators could become more competitive or even predominant in scale-free networks [13]. Fu found that, in small-world networks, the degree heterogeneity is critical to the emergence of cooperation; cooperation reaches its peak at some intermediate value of the fraction of hubs and not at either the most heterogeneous nor the most homogeneous case [10]. Moreover, many works have explored evolution dynamics over different configurations of different population structures [19], [20], [21], [22], [23], [24], [25].

There are common parameters in a graph that may have an impact on the emergence of cooperation, such as the average degree [10], [15] and the clustering coefficient [16], [17]. Tang and his colleagues discussed the role of connectivity (average degree) on the evolution of cooperation. Their experiments show that the cooperation rate of evolution has maximum value within a certain average degree and follows a one-peak function [15]. Ren examined the evolutionary dynamics on small-world networks with different levels of randomness, and found that the randomness can promote cooperation in a resonance-like way [26]. Wu et al. suggested that the resonance-like phenomena is actually related to the clustering coefficient instead of the small-world effect, as it is also exists in a regular ring graph, which does not exhibit the small-world effect [16]. Also, Lei et al. believe that the cooperation rate will only increase when the hubs are occupied by cooperators in a graph exhibiting a high clustering coefficient [17].

There has been some work in examining the role of the clustering coefficient in this domain [16], [17]. However, their main focus is to model real world networks, and hence the clustering coefficient, number of nodes, edges and degree are not controlled. Although the experiments are sufficient to explore the emergence of cooperation on these graphs, they do not fully explain the effect of the clustering coefficient in isolation of other parameter's influence. In this paper, we attempt to explore the effect of the clustering coefficient by fixing the other graph parameters as much as possible. We then compare the experimental results of the evolution of cooperation on a series of graphs with the same size (and the same number of edges if necessary), over a range of clustering coefficient values. Hopefully these results will help show the effect that clustering coefficient has on the emergence of cooperation.

Our approach has three main steps:

1) Create a graph generation algorithm that can generate graphs that satisfy all the requirements for our experiments.
2) Create the graphs and analyse mathematically the relationship between the graph structure and the clustering coefficient and attempt to predict the outcomes of simulations using this analysis.
3) Run the simulation experiments and observe the results. Compare the observed results with our predictions.

III. GENERATE GRAPHS WITH ADJUSTABLE CLUSTERING COEFFICIENT

The clustering coefficient can be calculated for a given node as the number of its neighbours that are connected over the total number of potential connections between them. Therefore, for the entire graph, the clustering coefficient can be calculated from the average of the clustering coefficient of each node (which is called the average clustering coefficient), or the number of triangles over the number of open triplets in the graph (also called transitivity). They both represent the graph's clustered architecture, with a little difference on the scaling of the value. However, the most common approach is using the transitivity (or the global clustering coefficient).

Most of the previous approaches to generating graphs with a tunable clustering coefficient attempt to model real-world networks which have the following common properties [27]:

1) Skewed degree distribution.
2) Low mean geodesic distance.
3) High clustering coefficient.

Many of the existing graph generation algorithms with tunable clustering coefficients are based on the preferential attachment model (where a higher degree vertex has a higher probability of attaching to new neighbours [28]) to generate graphs exhibiting a power law in node degree [27], [29], [30]. However, those approaches are only suitable for power law graphs with relatively low clustering coefficients. Several graph generation algorithms [27] are inspired by the Newman algorithm [31], which is based on the configuration model [32]. Experiments show that the clustering coefficients of the graphs generated by those algorithms may have increasing errors as the average degree increases [27]. There are also other algorithms that use degree sequence [27] and random walks [33].

In attempting to build a graph that closely models real-world graphs, most of the algorithms mentioned above can not guarantee a connected graph with a minimum error on the clustering coefficient. In this work, in addition to generating graphs with a tunable clustering coefficient, we also need the generated graphs to satisfy the following conditions:

1) Both the clustering coefficient and the graph size can be controlled.
2) The precision of the clustering coefficient is controllable (within a pre-defined error).
3) All of the vertices of the graph should be connected, i.e., there are no isolated sub-graphs.

Considering all of the above conditions, we proposed 2 new graph generation algorithms in this work.

A. Ascending graph generate algorithm (ascGen)

Given that we wish to generate a graph with N vertices and the desired total clustering coefficient is pc, let the actual clustering coefficient be denoted as ac, and the allowed error denoted as e, the pseudo-code of the ascGen algorithm is show in Algorithm 1

Note that there are some mechanisms adopted to prevent the generation to enter infinite loops.

Algorithm 1 Ascending graph generate algorithm

1. Create a fully connected graph (with a clustering coefficient = 0).
Create a graph with 3 vertices V_0, V_1, and V_2.
Add 2 edges to the graph, linking V_0 and V_1, and linking V_1 and V_2.
while $i < N$ **do**
    Add a new vertex V_i to the graph.
    Randomly link V_i to an existing vertex V_r, $(0 <= r < i)$.
end while
2. Keep adding edges (which can guarantee the addition of new triangles), until the actual clustering coefficient is close enough to the expected clustering coefficient.
while $pc - ac > e$ **do**
    Randomly pick a vertex V_n.
    if V_n has two unconnected neighbours V_p and V_q **then**
        connect V_p and V_q.
    end if
end while

The ascGen algorithm can generate the desired graph with a fixed number of nodes and a specified clustering coefficient in $O(V^3)$ time [1]. However, this approach has several drawbacks:

1) The number of edges in the graph increases until the desired clustering co-efficient is found. Hence, it is impossible to control the number of edges.
2) For graphs that have the same number of nodes, the clustering coefficient could be the same and yet have a different number of edges. As the edge that is added each time is not guaranteed to add new triangles (and hopefully increase the clustering coefficient), the graph that ascGen generates always has a relatively low number of edges (but not guarantee to have the minimum number of edges).
3) When the number of edges in the graph is low, each time we add an edge to create new triangles, the clustering coefficient increases, but when the number of edges is relative high in comparison to the number of nodes, each time we add a new edge, it is highly possible to add many more triplets, so, the clustering coefficient may decrease for a number of iterations as the edge number increases.

As we may create graphs with a desired clustering coefficient and varying number of edges (and hence average degree), we cannot explore the effect of clustering coefficient alone with these graphs. Another algorithm is required to create graphs of a fixed size, a specified clustering coefficient and a fixed number of edges.

B. Heuristic graph generation algorithm

Given we wish to generate a graph with N vertices, E edges, and a desired total clustering coefficient, pc, let the actual number of edges be denoted as e, and let the actual clustering coefficient be denoted as ac, and let the allowed

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which is similar to the time complexity to calculate the clustering coefficient

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error be denoted as e, the pseudo code of the heuristic graph generation algorithm is presented as Algorithm 2

Algorithm 2 Heuristic graph generate algorithm

1. Create a fully connected graph (with clustering coefficient = 0)
Create a graph with 3 vertices V_0, V_1, and V_2.
Add 2 edges to the graph, linking V_0 to V_1, and linking V_1 to V_2.
while $i < N$ **do**
    Add a new vertex V_i to the graph.
    Randomly link V_i to an existing vertex V_r, $(0 <= r < i)$.
end while
2. Add the rest of the edges on the connected graph randomly to generate a connected random graph with expected size and degree
while (**do** $e < E$)
    Add a new edge randomly
end while
3. Keep removing and adding new edge until the actual clustering coefficient is close enough to the expected clustering coefficient
while $pc - ac > e$ **do**
    for $i = 0; i < E; i + +$ **do**
        if the edge is not the only path between its two vertices **then**
            Calculate the potential clustering coefficient if this edge is deleted
        end if
    end for
    Delete the edge which produces a new clustering coefficient closest to the expected clustering coefficient following deletion
    for $i = 0; i < N; i + +$ **do**
        for $j = 0; j < N; j + +$ **do**
            if Node i and Node j are not connected **then**
                Calculate the potential clustering coefficient if we connect Node i and Node j
            end if
        end for
    end for
    Connect the two nodes which produce a new clustering coefficient closest to the expected clustering coefficient following connection
end while

IV. GRAPH PROPERTY AND THE EMERGENCE OF COOPERATION

A. Graph architecture

Given two graphs with the same number of nodes and edges, we may generate graphs with different clustering coefficients. Clustering coefficients can range from zero (e.g a tree structure) to one (a complete graph).

We adopt the heuristic graph generation algorithm to generate several graphs exhibiting the highest clustering coefficient with a constraint on both the number of vertices and the number of edges. The generated graphs with the maximum clustering coefficient had similarities in structure;

the maximum clustering coefficient was achieved with graphs containing a set of complete sub graphs each of the same size where each of the subgraphs was connected to other subgraphs by an edge (a bridge). A sample graph is depicted in Figure 1.
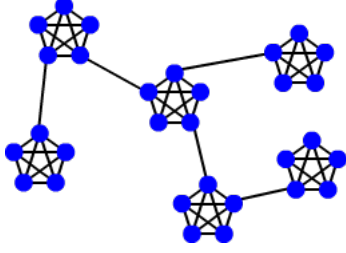

Fig. 1. The graph combined by 6 complete sub graphs of size 5

For a complete graph with N vertices, the graph has $\frac{N \times (N-1)}{2}$ edges. The clustering coefficient of the graph will be 1 as we have the same number of triples as we do triangles [2]. By connecting two complete graphs each of size N, the new graph will have the number edges given by:

$$2 \times \frac{N \times (N-1)}{2} + 1 = N \times (N-1) + 1$$

The clustering coefficient can be calculated by as:

$$\frac{\frac{N \times (N-1) \times (N-2)}{2}}{\frac{N \times (N-1) \times (N-2)}{2} + 2 \times (N-1)} = \frac{N^2 - 2N}{N^2 - 2N + 2}$$

Furthermore, for M complete sub graphs each with N vertices linked together by single bridges (no vertex has been chosen to be a point on more than one bridge), when $M \to \infty$, the clustering coefficient is:

$$\lim_{M \to \inf} \frac{M \times \frac{N \times (N-1) \times (N-2)}{2}}{M \times \frac{N \times (N-1) \times (N-2)}{2} + 2 \times (N-1)} = \frac{N^2 - 2N}{N^2 - 2N + 4}$$

.

Therefore, the maximum clustering coefficient of a graph with a fixed known number of both vertices and edges can be estimated. For example, to calculate the maximum clustering coefficient for a graph with n vertices and an average degree of d, we know the maximum clustering coefficient is when the graph comprises several complete sub graphs connected by bridging links. A complete graph with size s, has an average degree of $s-1$. If many of those graphs are connected together, the average degree will be roughly equal to $\frac{(s-2) \times (s-1) + 2 \times s}{s}$ (ignoring the nodes with the extra link connecting the subgraphs).

Therefore, the maximum clustering coefficient of a graph with n vertices and an average degree, d, is approximately the clustering coefficient of a graph comprising $m = n/([d]+1)$ complete subgraphs each with size $[d]+1$, which has the clustering coefficient of roughly $\frac{d^2 - 2d}{d^2 - 2d + 4}$ when m is small, and roughly $\frac{d^2 - 2d}{d^2 - 2d + 2}$, when m is big.

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[2]Number of triangles is given by $C_N^3 \times 3 = \frac{N \times (N-1) \times (N-2)}{2}$

Conversely, if we wish to calculate the lowest possible clustering coefficient for a graph, we must consider a graph with the minimum number of triangles in the graph. We know the clustering coefficient of any graph is equal to 0, provided there are no triangles in the graph.

In terms of minimising the number of edges in a graph with a clustering coefficient equal to zero, the smallest possible structure of the graph is a rectangle. In order to build a graph with the maximum number of edges for a given fixed size of nodes, N, we can construct a regular graph comprising rectangles. (which means the maximum distance between two nodes is 2). In other words, for a graph with size N, with a maximum number of the edges and a clustering coefficient = 0, the graph will be a regular graph with an average degree equal to $\lfloor N/2 \rfloor$.

B. The robustness of cooperation

The previous sections describe how different graphs with different clustering coefficients, and maintaining the same number of vertices and edges, can be created. By analysing the robustness of cooperation on these different graphs, we can systematically explore the clustering coefficient's influence on the emergence of cooperation.

The analysis below shows how the structure of the graph influences cooperation levels.

Let's assume the game to be run on the graph is a $N \times 2$ player game with the following payoff matrix: $\begin{bmatrix} 1 & 0 \\ \beta & \alpha \end{bmatrix}$

The players learn from their neighbour who receive the highest payoff. If the game is a social dilemma, then the following constraint holds: $\alpha < 1 < \beta < 2$.

For a defector with neighbours who are all cooperators, the average payoff the defector receives is $\beta > 1$, which means, it will obtain a greater score than its cooperative neighbours and will therefore invade its cooperative neighbours. As for any cooperator with at least one non-cooperative neighbour, the average payoff it receives is less than 1. Moreover, if the degree of the graph is sufficiently high, defection can spread and occupy the entire graph in a few generation, as Figure 2.

However, if a defector also has other neighbouring defector, its payoff may be smaller than 1, and on some certain graph structures will be too small a payoff to spread. Such a graph structure includes the scenario where a cooperative neighbour has much fewer non-cooperative neighbours than cooperative neighbours; this can provide the cooperators with a chance to prevent the invasion of defectors.
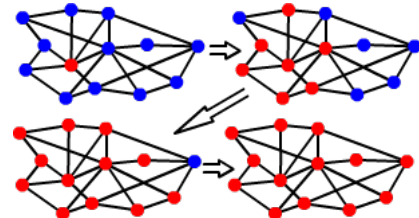

Fig. 2. The defectors invade the entire graph in a high degree graph

If the defector has n neighbours in total and m of them are also defectors, this defector's average payoff will be $(n-m)\beta + m\alpha$. For a neighbour of this defector, who has p neighbours but has only q defecting neighbours (including the defector it self), this neighbour will have the average payoff $p - q$.

Therefore, a defector can only invade a cooperative neighbour when $(n-m)\beta + m\alpha > p - q$.

It is highly possible that $p-q > (n-m)\beta + m\alpha$, especially when those two agents do not have too many common neighbours. However, if one player in a fully cooperative complete sub graph (the size of the sub graphs are n) mutates to become a defector, all of its direct neighbours will learn to defect by the next generation.

However, on the bridge between two subgraphs, a defector will have $m = n - 1$ defecting neighbours, so it will receive only $\beta + (n-1)\alpha$ as a payoff, which is much smaller than the payoff of its cooperative neighbour, who obtains a $n - 1$ payoff due to its $n - 1$ cooperative neighbours in its own complete subgraph. So, in this case, the defector will never invade another sub graph (Figure 3).

However, if the mutation happens to an agent on the bridge, it may invade 2 sub graphs, but no more (Figure 4).
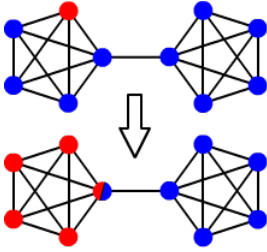

Fig. 3. one defector in the sub graph, since the vertex on the bridge linking to the other sub graph will get less payoff than the cooperator in the other sub graph that connected to it, it will continue swapping its strategy between cooperate and defect, and never invade other sub graphs.
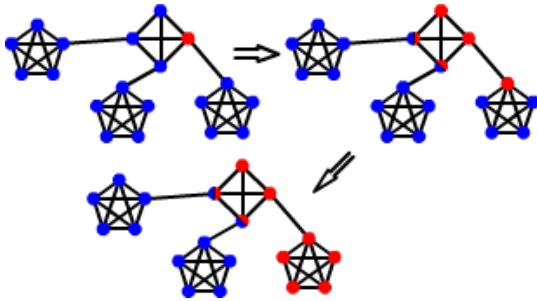

Fig. 4. The defector on the bridge may invade two sub graphs.

According to the analysis, the cooperation rate, following the introduction of one defector in the graph, will be less likely to be reduced as the clustering coefficient increases (while keeping both size and the number of edges constant). Therefore, we hypothesise that a higher clustering coefficient is beneficial to the robustness of cooperation, as the higher clustering coefficient leads to a more clustered graph.

However, the number of edges in the graph also influences the structure of the graph, so in order to fully explore the relationship between the clustering coefficient and the robustness of cooperation in the experiments, a fixed number of edges is needed.

V. EXPERIMENT RESULT

The game we used in this experiments is the prisoner's dilemma, with the following payoff matrix: $\begin{bmatrix} 1 & 0 \\ \beta & 0 \end{bmatrix}, \beta = 1.51$. This matrix has been used in much previous research.

In each generation, players will play a 2 player game against all of its direct neighbours and then receive the average payoff (as each player may have a different degree). They then learn the strategy that their most successful neighbour adopted. We adopt a simple learning model with no randomness so as to more easily understand the role each graph property plays in the evolution.

There are two main sets of experiments performed. First, we generate graphs with different clustering coefficients without controlling the number of edges, using the ascending graph generator. Second, we generate graphs with different clustering coefficients while guaranteeing a fixed number of edges.

The experiments will explore the influence of the clustering coefficient on the robustness of cooperation, and will also show whether the average degree influences the effect of the clustering coefficient on emergence.

A. Cooperation on graphs with different clustering coefficients without considering the average degree

The experiments are first undertaken on a set of graphs with 5000 vertices with the clustering coefficients varied from 0.1 to 0.6, generated by the ascending graph generation algorithm. The graph was initialized with all cooperators except for one random defector. We record the first 100 generations. Each experiment is the average value of 500 independent runs. The results are presented in Figure 5.
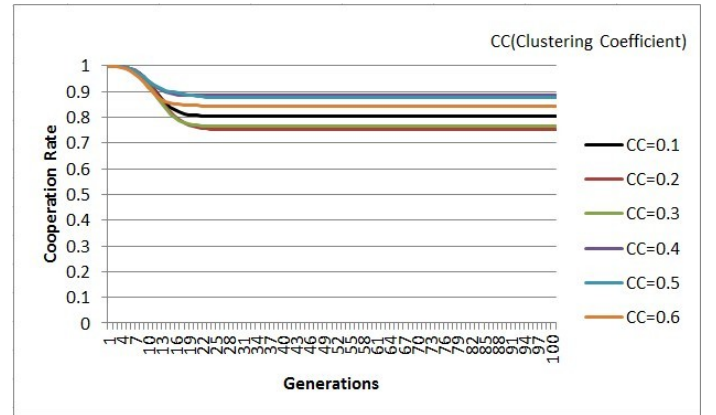

Fig. 5. without controlling the number of edges, the experiment result is not significant enough to show the influence of the clustering coefficient on the robustness of cooperation.

Figure 5 shows that when the clustering coefficient is equal to 0.1, 0.2, and 0.3, the cooperation rate is around 0.8, and

the cooperation rate increases to 0.9 when the clustering coefficient is equal to 0.4, 0.5, and 0.6. Despite some fluctuations, a general trend can be observed.

It may be because the ascending graph generation algorithm adds more edges when attempting to increase the clustering coefficient. With the addition of more edges, the clustering coefficients could increase without changing the inherent structure of the graph in to a highly clustered graph which can prevent the spread of the defection.

However, to explore the effect of the clustering coefficient on the emergence of cooperation, we not only need to use the same size graph in terms of the number vertices but same size graphs in terms of both the number of vertices and the number of edges.

B. Cooperation on graphs with different clustering coefficients taking into account the average degree

The experiments above showed that the cooperation rate will decrease quickly as the average degree of the graph increases. To fully understand the relationship of the clustering coefficient and the robustness of cooperation, another experiment has been undertaken which uses a set of graphs generated by the heuristic graph generate algorithm, with 1000 vertices, and a relative smaller number of edges, which is 2500, and different clustering coefficients. According to the calculation in the last section, the graph with the average degree of 5, will has the maximum clustering coefficient around 0.86 (Because the graph which has the architecture that is combined by a series of complete sub graphs with the size of 5, has the average degree 4.4, has the maximum clustering coefficient $\frac{5^2-2\times 5}{5^2-2\times 5+2} \approx 8.6$).To obtain a sufficient sample, 10 different graphs have been generated for each clustering coefficient, from 0.1 to 0.8. For each graph, the experiments has been ran 100 times independently. So, for each clustering coefficient, the result is the average value over 1000 independent runs. The result of the experiments is shown in the Figure 6
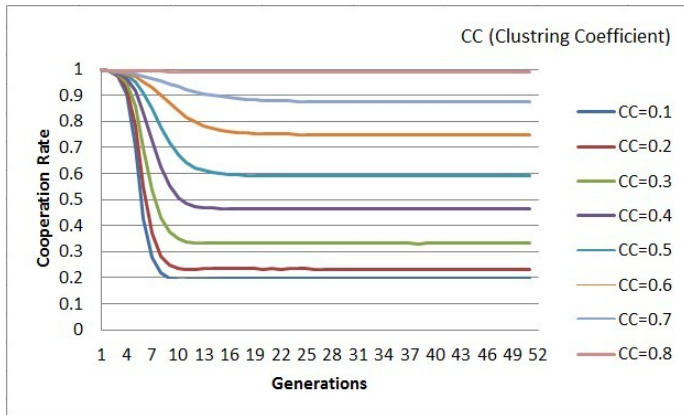

Fig. 6. With the same size, the cooperation rate of the graph is linearly increasing as the clustering coefficient increasing.

Figure 6 shows that with the same number of vertices and edges, the graph with high clustering coefficient, can benefit cooperation, prevent defector's spread.

VI. CONCLUSIONS

This paper presents two algorithms for generating graphs with a pre-defined clustering coefficient; the first constrains the number of numbers but not the number of edges; the second constrains both the number of nodes and the number of edges.

Simulations are used to explore how the average degree in the graph and the clustering coefficient of a graph influences the robustness of cooperation in a spatially organised population of agents.

The graph with a restricted number of edges and a high clustering coefficient is more likely to be constructed as a series of highly clustering subgraphs, which could, in theory, benefit cooperation. Moreover, the maximum clustering coefficient of a graph with a certain number of vertices and edges can be estimated. We can also calculate the maximum number of edges that can be added to the graph while maintaining a clustering coefficient as 0 in a graph of a specific size.

As we expected, experiments show that in a connected graph of cooperators, if the average degree is high, the co-operators are not robust if one player mutates to be a defector. Conversely, with a limited average degree, graphs with a high clustering coefficient can provide better cooperation.

Future research will investigate further into the graph topologies. This may include whether the multi-subgraph structure provides the best scenario for co-operators to be robust to invasion from defectors.

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